3.3 Linear Inequalities

Objectives

1) Solve linear inequalities
   - algebraically
   - graphically

- Still no interval notation in the Rockswold book, but you may use it instead (on quizzes and exams) so long as my instructions don't ask for a specific notation.

* Key detail: when isolating the variable, we do the operations as we would for solving a linear equation

EXCEPT

when we multiply both sides or divide both sides by a negative number, we must reverse the direction of the inequality

ex. ① $-2x < 3$

↓ divide by -2

$\boxed{x > -\frac{3}{2}}$

ex ② $-\frac{1}{2}x \geq 3$

↓ mult by -2

$\boxed{x \leq -6}$

Solve the inequality and write the solution using set notation.

a) algebraically

b) graphically

(3) $6 > x + 5$

a) isolate x by subtracting 5 from both sides

$1 > x$ "1 is greater than x-values"

reverse inequality when writing variable on the other side

$x < 1$ "x values are less than 1"

solution $\{x : x < 1\}$

b) $\underbrace{6}_{y_1} > \underbrace{x+5}_{y_1}$

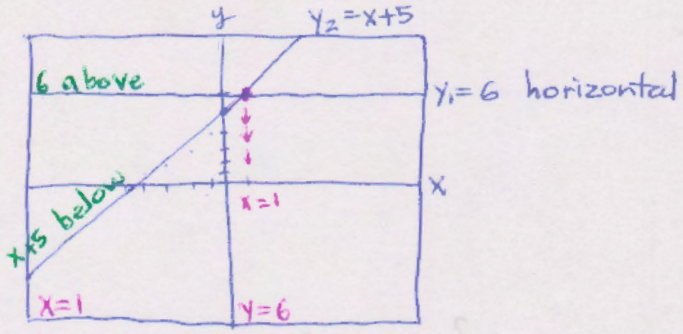
In GC [Y=]

$Y_1 = 6$

$Y_2 = X+5$

[ZOOM]

6. Standard window



Find intersection x-coordinate

2nd CALC

[TRACE]

5. Intersect

1st curve? [enter]

2nd curve? [enter]

Guess? [enter]

Identify the part of the graph where the "greater" equation is above the "lesser" equation.

Write x-values that correspond to the part of the graph that makes the inequality true.

$\{X : X < 1\}$

(4) $\frac{x}{3} \geq 2 + \frac{x}{6}$

a) algebraic method

option 1: clear fractions by multiplying equation by LCD.

$6 \cdot \frac{x}{3} \geq 6 \cdot 2 + 6 \cdot \frac{x}{6}$ cancel

$2x \geq 12 + x$ collect x's

$x \geq 12$

$\{x: x \geq 12\}$

option 2: Fraction lovers! Collect x by subtracting $\frac{x}{6}$ both sides

$\frac{x}{3} - \frac{x}{6} \geq 2$ $Y_3 - Y_6$ MATH ▸frac

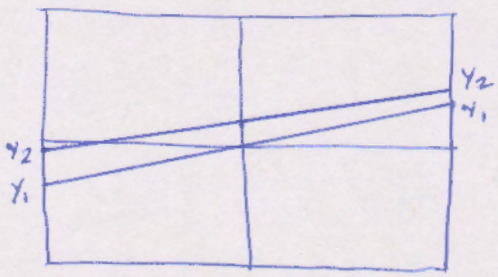
$\frac{x}{6} \geq 2$ mult by 2 both sides to isolate x

$x \geq 12$ isolate x

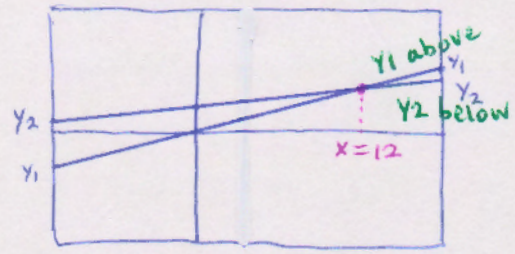
$\{x: x \geq 12\}$

b) graphical method

$\underbrace{\frac{x}{3}}_{Y_1} \geq \underbrace{2 + \frac{x}{6}}_{Y_2}$



WINDOW
XMAX 15
GRAPH



CALC
2nd TRACE
5. Intersect ENTER
ENTER
ENTER

$\{x: x \geq 12\}$

(5) $-5B + 2(B-1) \leq 6 - 5B - 1$

a) algebraic method

simplify first (LHS dist & combine, RHS combine)

$$-5B + 2B - 2 \leq -5B + 5$$

$$-3B - 2 \leq -5B + 5$$

collect all x's on the same side (add 5B both sides)

$$2B - 2 \leq 5$$

isolate x

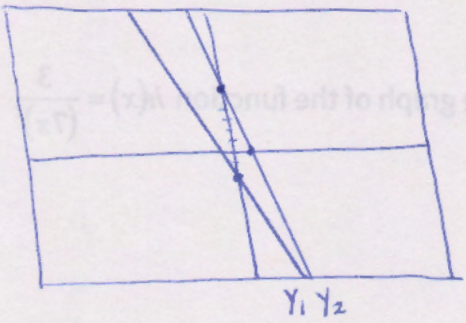
$$2B \leq 7$$

$$B \leq \frac{7}{2}$$

$$\left\{B : B \leq \frac{7}{2}\right\}$$

b) graphical method

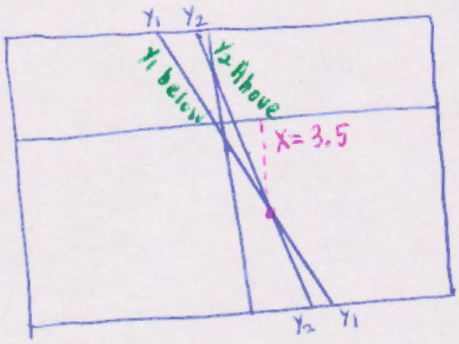
$$\underbrace{-5B + 2(B-1)}_{Y_1 = -5x + 2(x-1)} \leq \underbrace{6 - 5B - 1}_{Y_2 = 6 - 5x - 1}$$



WINDOW

XMIN −15 (less than −10)

GRAPH



CALC

2nd TRACE

5. intersect

ENTER

ENTER

ENTER

$\{x : x \leq 3.5\}$ but the question called it B...

$\{B : B \leq 3.5\}$

⑥ $6-2x \geq 4-2x$

a) algebraic

collect x's same side by adding $2x$ both sides

$6 \geq 4$

no x's left – but 6 is greater than 4, a true statement

all real #s

$\{x : x \text{ is a real \#}\}$

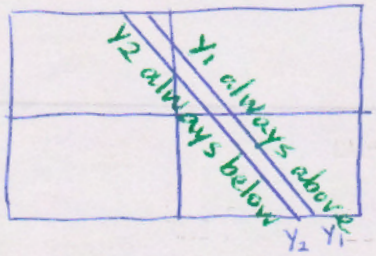
or $\{x : x \in \mathbb{R}\}$

$\in$ means "is an element of"

$\mathbb{R}$ means "the set of real numbers"

b) graphical

$\underbrace{6-2x}_{y_1} \geq \underbrace{4-2x}_{y_2}$



Both lines have slope $m=-2$ but different y-intercepts. Parallel lines do not intersect.

$\{x : x \in \mathbb{R}\}$

⑦ $6-x \leq 4-2x$

a) $6 \leq 4$ false statement

no solution

or ϕ the empty set

or $\{\}$ the empty set.

caution: NOT $\{\phi\}$ which is "the set containing the empty set".

b) see graph above

y_1 is Never above y_2.

no solution

(7) $-\frac{5}{8}x > \frac{25}{48}$

a) algebraic:

option 1: clear fractions, mult by LCD = 48

$48\left(-\frac{5}{8}x\right) > 48\left(\frac{25}{48}\right)$ cancel $\frac{48}{8} = 6$

$6(-5x) > 25$

$-30x > 25$

isolate x by dividing both sides by −30.
and change direction of inequality

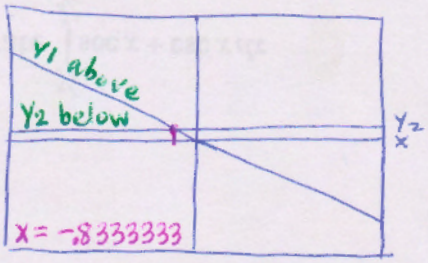
$x < \frac{25}{-30}$ reduce

$x < -\frac{5}{6}$

$\{x : x < -\frac{5}{6}\}$

b) graphical:

$\underbrace{-\frac{5}{8}x}_{Y_1} > \underbrace{\frac{25}{48}}_{Y_2}$



uh-oh! ☹
The Gc has given a rounded decimal answer, and this result (because it's inside a graph) won't convert to fraction with MATH ▸ frac.

check that the answer from the algebraic method matches:

$-5/6 = -.833333333333$ Ah, it does. ☺

$\{x : x < -\frac{5}{6}\}$ * always give an exact answer